Effect of Third Harmonic Generation on the Growth Rate of Raman Forward Scattering of an X-mode Laser Pulse in Magnetized Cold Plasma

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Nonlinear Raman forward scattering (NRFS) in the propagation of an X-mode laser pulse in a magnetized cold plasma channel is analyzed for the third harmonic scattered waves. 3rd harmonic nonlinear wave equation is set up to obtain the coupling equations and dispersion relation of excited upper-hybrid wave. Using Fourier transform and matching condition, the nonlinear growth rate of third harmonic NRFS is analytically calculated. It is shown that unlike the fundamental NRFS, the growth rate of third harmonic NRFS instability increases by increasing the external magnetic field.

Keywords: third harmonic NRFS, X-mode laser pulse, upper-hybrid wave, growth rate.

1 INTRODUCTION

Raman scattering is a parametric instability in which an incident light wave decays resonantly into an electron plasma wave and a scattered light wave [1]. Raman forward scattering (RFS) process in the transversely magnetized plasma includes the decay of an electromagnetic pump wave into an upper hybrid wave and two scattered Stokes/anti-Stokes sidebands. The laser and the sidebands exert the ponderomotive force on plasma electrons driving the excited upper hybrid wave. RFS is an important nonlinear process in laser beat wave and laser wake field accelerators [2]. It decreases the laser-plasma coupling during confinement fusion experiments so the nonlinear saturation of the RFS instability at high pump wave power is also a considerable problem in the high-energy laser-plasma interactions. In addition, RFS plays a dominant role in the evolution of the pulse in distances less than a Rayleigh length. It produces a plasma wave with large phase velocity (near the speed of light) that could accelerate charged particles to high energies [3].

Our purpose in this work is to find the growth rate of RFS instability in the presence of third harmonic generation in a transversely magnetized cold plasma when relativistic and ponderomotive nonlinearities are operative. Physically, this instability arises due to the coupling of density perturbation associated with the upper hybrid electrostatic wave and the oscillatory velocity of plasma electrons at $(2\omega_0, 2k_0)$. This coupling provides density perturbbation at the upper-hybrid frequency ω_{UH} shifted by the second harmonic frequency, which when beats with the velocity at ω_0 gives rise to $3\omega_0$ shifted by ω_{UH}. In the case of Raman shifted third harmonic forward scattering, nonlinear current density is generated at $(3\omega_0 \pm \omega_{UH})$ [4]. In order to study this phenomenon, we firstly find the nonlinear wave equation and the nonlinear equation of plasma motions in the laser radiation field to obtain the two coupled equations describing the third harmonic Raman scattering process. Based on our model, we solve the coupled equations to find the growth rate of RFS instability in the magnetized cold plasma. Finally, we investigate the variation of the growth rate with plasma density and static magnetic field.

2 3rd HARMONIC NONLINEAR WAVE EQUATION

Consider the propagation of an X-mode laser pulse with electric field $\boldsymbol{E}=E_0(\hat{x}+i\beta_0\hat{z})e^{i\theta_0}$ at (ω_0, k_0) in a plasma channel immersed in a static magnetic field $B\hat{y}$. Where,

$$\theta_0=(k_0 z-\omega_0 t), \qquad \beta_0=\frac{\omega_c}{\omega_0}\frac{\omega_p^2}{\omega_0^2-\omega_{UH}^2}$$

and ω_p, ω_c, ω_{UH} are the plasma frequency, electron cyclotron frequency, and upper-hybrid frequency, respectively [4]. Using the expansions of continuity equation and eqution of motion for plasma electrons, the nonlinear current density is obtained to set up the nonlinear wave equation,

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial z^2}\right) A_x = -\frac{4\pi n e^2}{m} \zeta_0 A_x \qquad (1)$$

$$\zeta_0 = a_0^2 \left(\frac{c^2 k_0^2 \omega_0}{4\left(\omega_0^2 - \omega_c^2\right)^4 \left(4\omega_0^2 - \omega_c^2\right)} N_1 - \frac{3}{8} \frac{\omega_0}{\left(\omega_0^2 - \omega_c^2\right)^4} N_2 \right) + \left(\frac{\omega_0^2 + \beta_0 \omega_0 \omega_c}{\omega_0^2 - \omega_c^2} \right)$$

And N_1, N_2 are defined by

$$\begin{aligned} N_1 &= 2\beta_0^2 \omega_0^7 + \beta_0 \left(6\beta_0^2 - 25\right)\omega_0^4 \omega_c^3 + 27\beta_0 \omega_0^2 \omega_c^5 \\ &+ \left(26\beta_0^2 - 5\right)\omega_0^3 \omega_c^4 - \left(30\beta_0^2 + 5\right)\omega_0^5 \omega_c^2 \\ &- \beta\left(6\beta_0^2 - 5\right)\omega_0^6 \omega_c + 10\omega_0 \omega_c^6 + \beta_0 \omega_c^7 \\ N_2 &= \left(3\beta_0^2 + 1\right)\left(\omega_0^7 + 6\omega_0^5 \omega_c^2 + \omega_0^3 \omega_c^4\right) \\ &+ 4\beta_0 \left(\beta_0^2 + 3\right)\left(\omega_0^6 \omega_c + \omega_0^4 \omega_c^3\right) \end{aligned}$$

Where, $a_0 (= eE/mc\omega_0)$ is the normalized potential vector. Through the Raman scattering process, the density perturbation beats with electron velocity at (ω_0, k_0) to generate the third harmonic nonlinear current density at $(3\omega_0 \pm \omega,\ k \pm k_0)$, and the wave equation for the third harmonic scattered waves is defined as

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) E_{(3\omega_0 \pm \omega)} = \frac{4\pi}{c^2}\frac{\partial}{\partial t} J_{x(3\omega_0 \pm \omega)} \qquad (2)$$

Here, the third harmonic current density has the form,

$$J_{x(3\omega_0 \pm \omega)} = \frac{i}{2} ec\left(n_0 a_3 \zeta_3 + \frac{1}{2} n_\omega \lambda_0 a_0 \zeta_0 \right) e^{i(3\theta_0 \pm \theta)}$$

$$\lambda_0 = -\frac{c^2 a_0^2 k_0^2 \left[\begin{array}{l} \left(\beta_0^2 + 4\right)\omega_0^2 \omega_c^2 + 2\left(\beta_0^2 - 1\right)\omega_0^4 \\ + 2\beta_0 \omega_0^3 \omega_c + 4\beta_0 \omega_0 \omega_c^3 + \omega_c^4 \end{array} \right]}{4\left(\omega_0^2 - \omega_c^2\right)^2 \left(4\omega_0^2 - \omega_c^2\right)}$$

where n_ω is density perturbation at frequency ω, a_3 is the amplitude of third harmonic scattered wave and ξ_3 is determined by replacing ω_0 with $(3\omega_0 \pm \omega)$ in the definition of ξ_0.

3 THE GROWTH RATE

Fourier transforming Eq. (2) gives the first coupled equation associated with the third harmonic Raman scattering governing the propagation of scattered Stokes and anti-Stokes waves with amplitude $A_{3\pm}$ in transversely magnetized plasma,

$$\left(\omega_{3\pm}^2 - c^2 k_{3\pm}^2 - \zeta_{3\pm}\omega_p^2\right) A_{3\pm} = \frac{2\pi e^2}{m} \zeta_0 \lambda_0 A_0 n_\omega \quad (3)$$

Now, using the perturbed form of the continuity equation and Lorentz force equation one can find the second coupled equation describing the coupling between fluctuations of the amplitude of scattered waves and plasma density perturbations,

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{UH}^2\right) n_\omega = \frac{n_0 e^2}{m^2 c^2} \frac{\partial^2}{\partial z^2} \left(\zeta_{3s} A_{3s} A_{3x} + \eta_{3s} A_{3s} A_{3z} \right) \qquad (4)$$

Using Fourier transform of Eq. (4), and looking for $e^{i(kz - \omega t)}$ dependence, then combining it with Eq. (3), gives the dispersion relation for the excited plasma wave during the 3rd harmonic Raman forward scattering,

$$\left(\omega^2 - \omega_{UH}^2\right) = \frac{c^2 k^2 a_0^2 \omega_p^2 \zeta_0 \lambda_0}{4} \left[\frac{\left(\zeta_{3+} - \eta_{3+}\beta_{3+}\beta_{30}\right)}{D_{3+}} + \frac{\left(\zeta_{3-} - \eta_{3-}\beta_{3-}\beta_{30}\right)}{D_{3-}} \right] \qquad (5)$$

with $D_{3-} = \omega_{3-}^2 - c^2 k_{3-}^2 - \zeta_{3-}\omega_p^2$. Here, D_{3-} and D_{3+} are dispersion coefficients related to the 3rd harmonic scattered Stokes and anti-Stokes waves, respectively.

In Raman forward scattering process, both scattered Stokes and anti-Stokes waves are simultaneously resonant with matching condition $\omega_{3\pm} = 3\omega_0 \pm \omega_{UH}$. So, the growth rate of third harmonic Raman forward scattering instability Γ_{NRFS} is obtained easily by putting $D_{\pm} = 2i\Gamma_{NRFS}\left(\omega_{UH} \pm \omega_0\right)$ in Eq. (5). Thus the growth rate turns out to be

$$\Gamma_{NRFS} = \frac{c k a_0 \omega_p \lambda_0 \left(\zeta_0 + \eta_0 \beta_0\right)}{\sqrt{8\left(\omega_0^2 - \omega_{UH}^2\right)}} \qquad (6)$$

Here, the value of k can be defined from the resonance condition for Stokes wave i.e. $D_{3-}(3\omega_0 - \omega_{UH}) = 0$,

$$k = 3k_0\left[1 - \sqrt{(1-\omega_{UH}/\omega_0)^2 - \zeta_0\,\omega_p^2/\omega_0^2}\right].$$

4 RESULTS

In Fig. 1, the variation of normalized growth rate of 3rd harmonic NRFS instability (Γ_{NRFS}/ω_0) has been plotted against the normalized plasma frequency (ω_p/ω_0) for $\omega_c/\omega_0 = 0.05$. It is illustrated that for a laser with high intensity, the growth rate is high. It is also shown that the growth rate increases with increasing the plasma density.

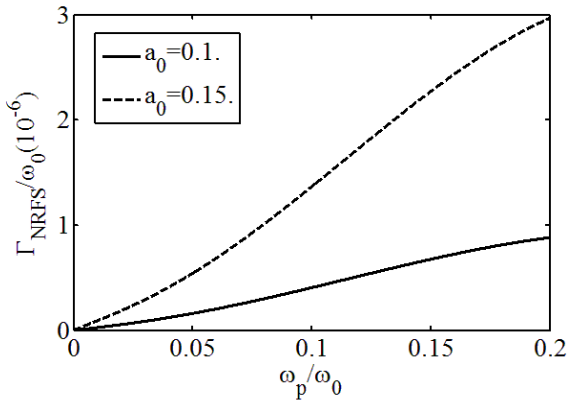

Fig. 1: The variation of normalized NRFS growth rate against the normalized plasma frequency

Fig. 2 shows the variation of the normalized 3rd harmonic NRFS instability against the normalized cyclotron frequency ω_c/ω_0 for $\omega_p/\omega_0 = 0.1$ and $a_0 = 0.15$. One can observe that the growth rate increases by enhancing the external magnetic field.

5 CONCLUSION

In this paper, we have considered the propagation of an extraordinary pump laser wave in transversely magnetized cold plasma and investigated the effect of third harmonic generation on the growth rate of nonlinear Raman forward scattering. Nonlinearity arises due to the ponderomotive and relativistic motions of plasma electrons and their coupling with plasma channel. Third harmonic Raman instability occurs due to the coupling of plasma density perturbation associated with the upper hybrid electrostatic wave and the oscillatory ve-

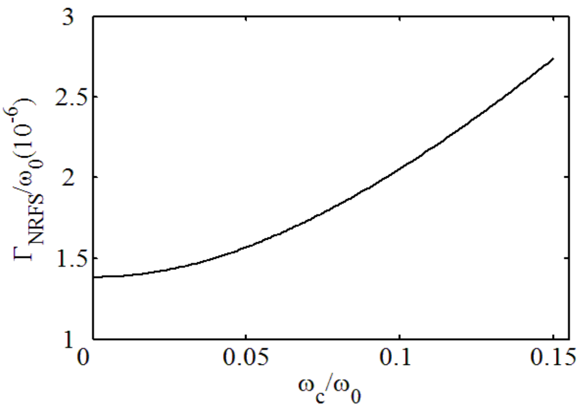

Fig. 2: The variation of normalized NRFS growth rate against the normalized cyclotron frequency

locity of plasma electrons at the third harmonic frequency. We have found an expression for the growth rate of third harmonic Raman forward scattering in transversely magnetized plasma. Comparing the results of the present study with that we obtained in the case of fundamental Raman instability [3], reveals that as the Raman instability switches into the third harmonic scattering, the value of its growth rate decreases approximately by the factor of $(1/100)$. In comparison with the third harmonic Raman backward scattering instability [4] in which the growth rate decreased by increasing the external magnetic field, in the case of third harmonic Raman forward scattering, this value increases by increasing the external magnetic field. The reason is that the presence of magnetic field increases the transverse quiver velocity and also leads to the generation of a longitudinal velocity component. This effect enhances the plasma fluctuations and hence the instability will grows.

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