Modelling Radiative Properties of SF$_6$ Arc Plasma

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The objective of this work is to compare the accuracy of several approximate radiative properties models for the prediction of radiative transfer in SF$_6$ arc plasma at the temperatures in the range of 300–25000 K and pressure of 1 MPa. Calculated absorption coefficients are used to generate the parameters of different approximate models. In order to compare the accuracy of various approximate models, the radiative transfer inside the cylindrically symmetrical SF$_6$ plasma with prescribed temperature profile was calculated using the P1-approximation.

Keywords: plasma radiation, SF$_6$ arc plasma, radiative property modelling

1   INTRODUCTION

Total energy balance in thermal plasmas is strongly influenced, or even dominated by radiative transport of energy. The radiative fluxes or the balance between emission and absorption of radiative power are very important for an understanding of the physical behavior of the arc plasmas.

Predicting the radiative transfer form an exact resolution of the spectral radiative transfer equation remains a formidable task because of the need to treat continuum radiation and hundreds of spectral lines spanning wavelengths from the far infrared to the short ultraviolet spectral regions. Among various approximate methods [1 - 4] the P1-approximation has extensively been used in association with different simplifications of radiative properties.

The aim of this work is to compare the accuracy of several approximate radiative properties models for the prediction of radiative transfer in SF$_6$ arc plasma at the temperatures in the range of 300 – 25000 K and pressure of 1 MPa. Calculated absorption coefficients are used to generate the parameters of different approximate models: (1) Planck MAC (Mean Absorption Coefficient), Rosseland MAC and their combination for two different splitting of frequency interval; (2) Planck MAC and Rosseland MAC calculated for frequencies grouped based on frequency and also on magnitude of absorption coefficients; (3) SLW (Spectral Line Weighted Sum of Gray Gases) model, combined with Planck MAC and Rosseland MAC. In order to compare the accuracy of various approximate models, the radiative transfer inside the cylindrically symmetrical SF$_6$ plasma with prescribed temperature profile was calculated using the P1-approximation. It is shown that good accuracy with exact spectral integration gives the combination of Planck and Rosseland MAC calculated for frequency and magnitude grouping.

2   ABSORPTION COEFFICIENTS AND APPROXIMATE MODELS

Absorptivity κ_ν is proportional to the concentrations of the chemical species occurring in the plasma. In SF$_6$ plasma, we assume following species: SF$_6$ molecules, S and F neutral atoms, S$^+$, S^{+2}, S^{+3}, F$^+$, F^{+2} ions and free electrons. Equilibrium composition of the plasma was computed using Tmdgas computer code [5].

Spectral absorption coefficients κ_ν were calculated using semi-empirical formulas to represent both continuum and line radiation. Continuum spectrum is formed by bound-free transitions (photoionization) and free-free transitions (bremsstrahlung). Photoionization cross sections for neutral atoms were calculated by the quantum defect method of Burgess and Seaton [6], the cross sections of photoionization of ions and free-free transitions were treated using Coulomb approximation for hydrogen-like species [7]. In the discrete radiation calculation spectral lines broadening and their complex shapes were considered. The lines are broadened due to numerous phenomena; the most important are Doppler broadening, Stark broadening, and resonance broadening. Calculated total spectral absorption coef-

ficients for SF$_6$ plasma at the pressure of 1 MPa and the temperatures of 5 000 K and 15 000 K are presented in Fig. 1.
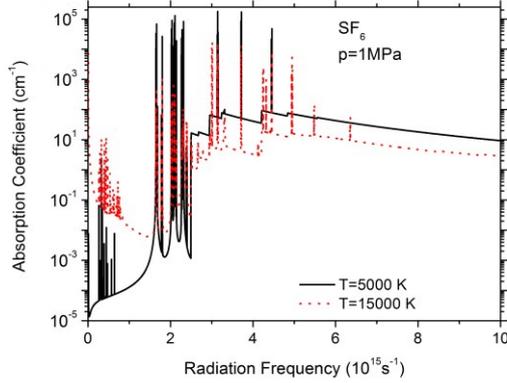

Fig. 1: Absorption coefficients of radiation for SF$_6$ plasma

2.1 PLANCK AND ROSSELAND MAC MODEL

Mean absorption coefficient (MAC) models use a splitting of the spectral range into several frequency groups across which the absorption coefficient is assumed to be constant and equal to a mean value $\overline{\kappa_\nu}$. The more frequency groups, the more accurate results we get. However, the number of groups should be minimized to decrease the computation time. In this work, the frequency interval $(10^{12} - 10^{16})$ s^{-1} was split into

(a) five frequency groups with splitting
  (0.001, 1, 2, 3.3, 8, 10)x10^{15} s^{-1}

(b) ten frequency groups with splitting
  (0.001, 0.151, 0.89, 1.585, 2.172, 2.488, 3.143, 4.117, 4.839, 8.441, 10)x10^{15} s^{-1}

The mean values of absorption coefficients were taken as either Planck (κ_P) or Rosseland (κ_R) means:

$$\kappa_P = \int_{\nu_k}^{\nu_{k+1}} \kappa_\nu B_\nu \, d\nu \Big/ \int_{\nu_k}^{\nu_{k+1}} B_\nu d\nu \,, \quad (1)$$

$$\kappa_R^{-1} = \int_{\nu_k}^{\nu_{k+1}} \kappa_\nu^{-1} \frac{dB_\nu}{dT} \, d\nu \Big/ \int_{\nu_k}^{\nu_{k+1}} \frac{dB_\nu}{dT} \, d\nu \,, \quad (2)$$

where B_ν denotes the Planck function. The five spectral bands and the mean absorption coefficients for temperature of 15 000 K are shown in Fig. 2 with the high resolution spectra. The temperature dependence of mean absorption coefficients for five groups splitting is shown in Fig. 3.
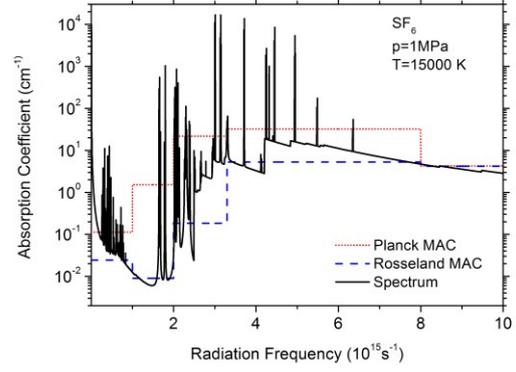

Fig.2: The high resolution spectrum of SF$_6$ plasma compared with Planck and Roseland MAC
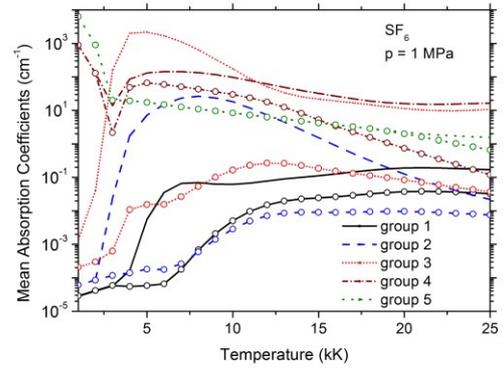

Fig. 3: MAC as a function of temperature for five groups splitting: lines – Planck MAC, lines + symbols - Rosseland MAC

It can be seen that Planck MACs overestimate the influence of spectral lines, on the other hand Rosseland MACs underestimate it. Great difference between Planck and Rosseland MAC occurs especially for groups 2 and 3 (i.e. for frequencies from $(1$x$10^{15} - 3.3$x$10^{15})$ s^{-1}).

2.2 κ-BASED GROUP AVERAGING

To improve the accuracy of the group averaging method 2.1, further splitting of particular frequency groups to subgroups based on the magnitude of κ was proposed in [8]. The frequency interval was split into 8 frequency groups

(0.001, 1, 1.4, 1.77, 2, 2.2, 2.5, 3,10)x10^{15} s^{-1} .

Every frequency group was divided to 4 subgroups based on the magnitude of κ_ν (in cm^{-1})

$\kappa_\nu \in \{0, 0.1\}, \{0.1, 1\}, \{1, 10\}, \{10, \infty\}$.

Planck and Rosseland MACs were calculated for each subgroup.

2.3 SLW MODEL

The Spectral Line Weighted Sum of Gray Gases (SLW) model has been developed by Denison and Webb [9]. In this model the absorption coefficient domain is divided into several ranges m (in our case $m = 10$), bounded by k_{j+1} and k_j, $k_{j+1} > k_j$. The lower (k_1) and upper (k_m) bounds of the domain partition are chosen such that k_1 is smaller than minimum κ_v, and k_m is greater than maximum κ_v. The absorption coefficient boundaries k_j are obtained by equally partitioning the absorption coefficient domain in a logarithmic scale. For each absorption coefficient range j, associated frequency ranges $\Delta v_{j,i}$ are defined such that for each v in $\Delta v_{j,i}$ the actual absorption coefficient κ_v lies in the range $\{k_j, k_{j+1}\}$ (Fig. 4).

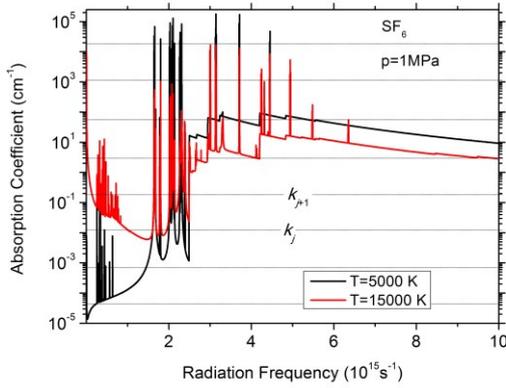*Fig. 4: SLW absorption coefficients domain partition*

Representative constant absorption coefficient values within each range are chosen as:

(a) $\overline{\kappa_j^*} = \sqrt{k_j k_{j+1}}$
(b) Planck mean, i.e. $\overline{\kappa_j} = \kappa_{Pj}$
(c) Rosseland mean, i.e. $\overline{\kappa_j} = \kappa_{Rj}$

The radiation transfer equation for each coefficient range can be written as

$$\vec{\Omega} \cdot \nabla I_j = -\overline{\kappa_j}\left(I_j - B_j\right), \qquad (3)$$

where $\vec{\Omega}$ is the unit direction vector and the weights B_j

$$B_j = \sum_i \int_{\Delta v_{j,i}} B_v dv . \qquad (4)$$

3 P1-APPROXIMATION

In order to compare the accuracy of various approximate models, the radiative transfer inside the cylindrically symmetrical SF$_6$ plasma with prescribed temperature profile was calculated using the P1-approximation. The P1-approximation consists of expanding radiative intensity in spherical harmonics and including only the first order terms. Under this assumption the equation of radiative transfer leads to simple elliptic equation for the group density of radiation U_j

$$\nabla \cdot \left[-\frac{c}{3\overline{\kappa_j}}\nabla U_j\right] + \overline{\kappa_j}cU_j = 4\pi B_j \overline{\kappa_j} . \quad (5)$$

The group radiative flux $\vec{F_j}$ is given by

$$\vec{F_j} = -\frac{c}{3\overline{\kappa_j}}\nabla U_j . \qquad (6)$$

The net emission of radiation is then

$$\nabla \cdot \vec{F_R} = \sum_j \nabla \cdot \vec{F_j} . \qquad (7)$$

Figures 5 – 8 compare the net emission calculated with various average absorption coefficients with results of the exact spectral integration [8]. In Fig. 5, MACs for 5-groups frequency splitting were used; Fig. 6 shows results of MACs for 10 frequency groups. Results of κ-based group averaging are shown in Fig. 7. In Fig. 8, the spectral averaging based on SLW model is presented.

It can be seen that Planck MAC overestimate the net emission in all cases. The combination of Planck and Rosseland MACs was carried out such that Planck MAC were considered in frequency groups with low values of κ_v, in groups with high values of κ_v Rosseland MAC were used. The best fit with exact spectral integration was achieved using combination of Planck and Rosseland MACs calculated for κ-based averaging.

4 CONCLUSIONS

Comparison of the net emission in SF$_6$ plasma predicted using P1-approximation with various approximate radiative models and exact spectral integration have been performed. Planck MAC overestimate both emission and absorption of radiation for all used radiative models. Good accuracy with exact spectral integration gives the combination of Planck and Rosseland MAC calculated for frequency and magnitude grouping.
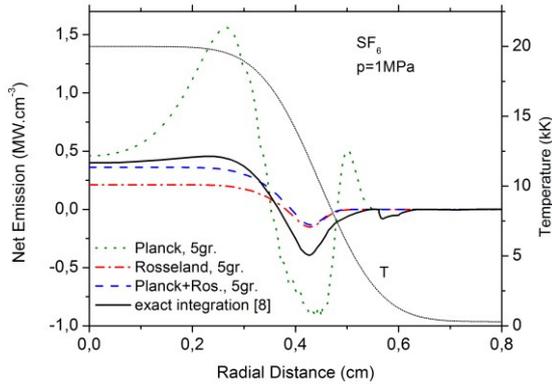
Fig. 5: Comparison between net emission obtained from exact spectral integration [8], and from Planck or Rosseland MAC for 5-groups frequency splitting
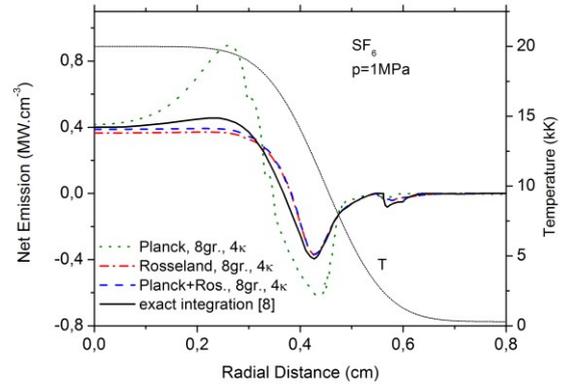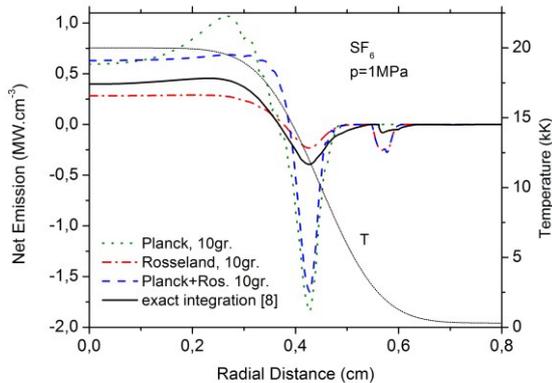

Fig. 6: Comparison between net emission obtained from exact spectral integration [8], and from Planck or Rosseland MAC for 10-groups frequency splitting
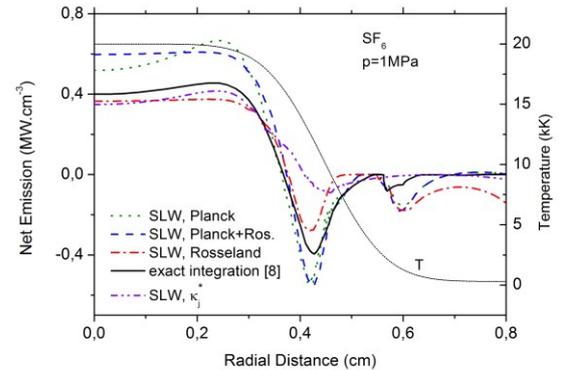

Fig. 7: Comparison between net emission obtained from exact spectral integration [8], and from Planck or Rosseland MACs for κ-based spectral averaging

Fig. 8: Comparison between net emission obtained from exact spectral integration [8], and from SLW model

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