QUANTIFYING UNCERTAINTIES IN MEAN ABSORPTION COEFFICIENTS FOR A WALL-STABILIZED ELECTRIC ARC

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Abstract. The quality of arc simulations depends significantly on radiation modeling. Uncertainties due to physical parameters and modeling errors should be systematically quantified. We solve the energy balance equation for a wall-stabilized arc using the P1 model, i.e. without a prescribed temperature profile. We derive the linearized equation and assess the arc voltage sensitivity. This method allows us to optimize the definition of mean absorption coefficients consistently and at low computational costs.

Keywords: sensitivity, mean absorption coefficient, P1 model, optimization, arc simulation.

1. Introduction

Electric arcs and their numerical modeling in industrial devices lead to a diverse set of questions raising many separate research topics. One of them is radiation, since electrical arcs dissipate enormous amounts of thermal energy leading to elevated temperature levels. Therefore, radiation is almost always an important energy transfer mode in electrical arcs.

The basic relations of radiation are given by the radiative transfer equation (RTE)

\[ \hat{s} \cdot \nabla I_\nu = \kappa_\nu (B_\nu - I_\nu), \]

which is stated here for the simplest case of a non-scattering medium in local thermal equilibrium (LTE).\[1\] It describes the change of radiative intensity \( I_\nu \) along direction \( \hat{s} \) that is due to emission, described by the Planck function \( B_\nu \) and given in equation \[5\] below, and absorption of the medium. Solving the RTE is computationally demanding since the radiative intensity is a function of location, direction, and frequency. The absorption coefficient \( \kappa_\nu \) and its impact on the computational complexity of the RTE are discussed subsequently.

To meet the limited computer resources available to application engineers, the complexity of the RTE must be reduced to simpler models that account appropriately for the radiative thermal energy transport. The method of net emission coefficients (NEC) \[2\] is often used and has been developed to acceptable accuracy \[3\]. Its main advantage lies in the marginal computational costs incurred at run time of a plasma simulation, since the radiative thermal source term is tabulated in advance. However, evaluating the radiative heat flux on a nearby surface requires more sophisticated methods. One option is the P1 model, which is a first-order approximation of the RTE in spherical harmonics and frequently used in industrial arc simulations. This model leads to additional differential equations that are of the same structure as the conservation equations for mass, momentum, and energy.

We also have to consider the material data. The most important quantity in radiation modeling is the spectral absorption coefficient \( \kappa_\nu \), which depends on the radiation frequency \( \nu \), gas pressure, temperature, and gas composition. The complex structure of the absorption spectrum (see figure \[1\]) is due to the atomic structure of the gas (see, e.g., \[1\]). It does not allow for a naive usage of this data, since an appropriate resolution of the frequency domain would lead to excessive computational costs. Despite the large variations in the value of the absorption coefficient, the frequency domain is split into a small number of intervals or bands and mean absorption coefficients (MAC) are computed for each of them. Since each frequency band acts as a gray body, this approach is termed as the multi-banded gray P1 model.

The main problem of the averaging step is how the absorption lines shall be treated that span over multiple orders of magnitudes. Classically, the Planck and Rosseland average are considered which are weighted...
averages of $\kappa_i$ and $\bar{\kappa}_i$, respectively. The former is dominated by the peak values of the absorption lines while the latter almost ignores them. It is clear that the MAC depend on the definition of the frequency band boundaries that are often chosen heuristically.

The raw data itself comes with uncertainties that affect the MAC values and propagate to the final results. Their sensitivities can be quantified by running simulations repeatedly with slightly edited input data, but comes with major computational costs. For small changes in the MAC, however, we will show that the same information can be obtained much quicker by deriving the linearized equations and solving for the first-order effects on the arc voltage. Hence, we split the spectral domain into a finite number of intervals $D_i = [\nu_{i-1}, \nu_i]$. The mean absorption coefficients $\kappa_i = \langle \kappa \rangle_i$ are obtained using the Planck and Rosseland average

$$\langle \kappa \rangle_i^D = \frac{\int_{D_i} B_\nu \kappa \, d\nu}{\int_{D_i} B_\nu \, d\nu}, \quad \langle \kappa \rangle_i^R = \frac{\int_{D_i} B'_\nu \, d\nu}{\int_{D_i} B'_\nu \kappa^{-1} \, d\nu},$$

respectively. Therein, $B_\nu(T)$ denotes the Planck function

$$B_\nu(T) = \frac{2h}{c^2} \frac{\nu^3}{\exp(h\nu/kT) - 1},$$

$B'_\nu$ is its derivative with respect to temperature, $c$ denotes the speed of light in vacuum, and $h$, $k_B$ are the Planck and Boltzmann constants, respectively. The P1 model approximation results in an expression relating the radiative heat flux $\tilde{F}_\nu$ and the irradiation function $G_\nu$ by:

$$\tilde{F}_\nu = \frac{-1}{3\kappa_i} \nabla G_\nu.$$}

This also holds for band-averaged quantities, so that $G_i$ is obtained as the solution of the linear problem

$$\mathcal{L}_i(G_i) := \text{div} \left( \frac{-1}{3\kappa_i} \nabla G_i \right) + \kappa_i(G_i) = \kappa_i 4\pi B_i(T)$$

with $B_i(T)$ denoting the band-integrated Planck function. The radiative heat sink is then given by

$$U = (-1) \sum_i \kappa_i (4\pi B_i(T) - G_i),$$

which follows from the RTE \cite{1} by integration over the solid angle.

2.2. Linearized equation

Equations (2) and (8) show that any change in the absorption coefficient results in changes to the temperature profile. For small variations, we can derive the linearized energy conservation equation with respect to increments in the MAC $\delta \kappa_i$ and temperature $\delta T$. This first-order expansion provides a relation which we write as

$$C_{th}(\delta T) - M_T(\delta T) = \delta U,$$

with $\delta U = \delta U(\delta \kappa_i, \delta T)$ summarizing the linear effects in the radiation model. The details are derived in the remainder of this section.

The linearized heat conduction is given by

$$C_{th}(\delta T) = \text{div} \left( -\lambda(T) \nabla \delta T \right),$$

with $\lambda(T) = \frac{d}{dT} \lambda(T)$. The linearized Ohmic heating at constant current results in
We use a domain radius of \( R = 5 \text{ mm} \) discretized with 200 cells and cell-centered quantities. The wall temperature is fixed at 300 K. The energy conservation equation \((11)\) is iteratively solved using a relaxation update with a constant current of \( I = 50 \text{ A} \) until the maximal temperature update is smaller than 1 K.

The radiation frequency bands are defined as \( N \) uniformly spaced intervals \( D_i = [\nu_{i-1}, \nu_i] \), \( i = 1, \ldots, N \), with \( \nu_0 = 10^{10} \text{ Hz} \) and \( \nu_N = 6 \times 10^{15} \text{ Hz} \). An additional frequency band is defined \( D_{N+1} = [\nu_N, 10^{16} \text{ Hz}] \). Material data and spectral absorption coefficients are taken for air at \( p = 10 \text{ bar} \). \(2\)

\[
M_t(\delta T) = \left( \sigma'(T) - 2 \frac{\sigma(T) \delta S[T]}{S[T]} \right) \frac{I^2}{S[T]} \delta T \tag{11}
\]

where \( \frac{\delta S[T]}{S[T]} \) represents the functional derivative of the conductivity, \( \sigma'(T) = \frac{d}{dT} \sigma(T) \), and the radiative heat sink increment is expanded to

\[
\delta U = (-1) \sum_i (4\pi B_i(T) - G_i) \delta \kappa_i + \kappa_i (4\pi B_i'(T) \delta T - \delta G_i) . \tag{12}
\]

The linear increment in the irradiation function \( \delta G_i \) is obtained as the solution of

\[
\delta G_i = L^{-1} \left( M_{\kappa,i} \delta \kappa_i + M_{th,i} \delta T \right) \tag{13}
\]

with

\[
M_{\kappa,i}(\delta \kappa_i) = \text{div} \left( \frac{-1}{3\kappa_i^2} \text{grad} (G_i) \delta \kappa_i \right) + (4\pi B_i(T) - G_i) \delta \kappa_i , \tag{14}
\]

\[
M_{th,i}(\delta T) = \kappa_i 4\pi B_i'(T) \delta T . \tag{15}
\]

In summary, the terms can be rearranged so that the increment in the temperature profile is given as a linear function of those in the MACs,

\[
\delta T = Z^{-1} \sum_i Y_i \delta \kappa_i , \tag{16}
\]

with \( Z \) and \( Y \) being linear operators given by

\[
Z := C_{th} - M_t + \sum_i \kappa_i 4\pi B_i'(T) - \kappa_i L^{-1} M_{th,i} , \tag{17}
\]

\[
Y_i := \kappa_i L^{-1} M_{th,i} - (4\pi B_i(T) - G_i) . \tag{18}
\]

Finally, we find an expression for the linear increment in the electric field with respect to a temperature increment:

\[
\delta E = (-1) \frac{I}{S[T]} \frac{\delta S[T]}{\delta T} \delta T . \tag{19}
\]

## 3. Results and Discussion

Figure 2 shows temperature profiles of the wall-stabilized arc obtained with Planck and Rosseland average using 8 bands. We also plot the reference solution with \( N = 65536 \) bands, which shows an arc center temperature of 10970 K and a noticeable core region with a higher temperature gradient than in the outer parts. Near the wall, the temperature drops quickly to the prescribed wall temperature. In comparison to this, the Rosseland average leads to a more pronounced core region with a higher arc center temperature but qualitatively similar temperature profile. In contrast, the Planck average shows a diffusive temperature profile with an arc center temperature much lower than the reference solution, and a sightly increased temperature in the outer parts. These differences disappear gradually with finer spectral resolutions and the temperature profiles converge to the reference solution.

This data is explained by the properties of the averaging methods. Since the Planck average is dominated by the peak values of the absorption lines, it resem-
bles an optically thick plasma, and the radiative heat transfer has the same effect as an increased thermal conductivity. On the other hand, the Rosseland average yields an optically thin material and radiative energy is transported to the walls.

Figure 3 shows the electric field in variation of the spectral resolution and for several arc currents. We see that, at low spectral resolutions, the Planck average yields a higher electric field than the Rosseland average. This is due to the lower temperature levels and lower electrical conductivity. We also note that the curves converge to the reference solution, with the Planck average being rather constant for spectral resolutions \( N > 2^{10} \). This figure shows that accurate results are only obtained with a finely resolved absorption spectrum.

We now turn to the question, which parts of the spectrum are most significant. Figure 4 and 5 show the relative sensitivity of the electric field obtained by the linearized equation for the two averaging methods. We used an uncertainty of 1% in the MAC for each band individually. The same data is also obtained with direct computations for \( N \leq 256 \), i.e. editing the MAC values and running the simulation until convergence, but at much higher computational costs. The results coincide almost exactly and validate the linearized method. Only minor differences are noted for the solution using \( N = 8 \) bands and Rosseland average: however, they are at higher radiation frequencies where the sensitivities are orders of magnitudes lower and therefore negligible. This method has also been applied to finely resolved spectral data with drastically reduced computational effort.

The sensitivity of the electric field is limited by 1\% for the 8-banded solution. The sensitivity curves scale with the interval length; in fact, considering the relative sensitivity of the electric field per interval length, i.e. \( \Delta E/\Delta E_{\text{ref}} \), results in a characteristic curve. We also see that a higher number of spectral bands allows to resolve the sensitivities of the absorption lines. The sensitivity curve of a finer resolved solution is bounded almost everywhere by the coarser ones. The Planck average yields notable lower sensitivities at frequencies above \( 2.5 \times 10^{15} \) Hz and band resolutions \( N \leq 256 \).

4. Conclusions

We presented a simple model to study effects of band-averaged MACs on the temperature profile and the electric field of a wall-stabilized arc. The linearized method provides a systematic methodology to assess the sensitivity of the frequency bands at low computational costs, and helps to define MAC inside each band. Hence, frequency bands of low sensitivity can be identified and may be merged or grouped into coarser ones. For future work, the method is easily extended to non-uniform intervals. Preliminary tests with a renormalization length \( \Delta \) are promising and are currently being investigated.

Acknowledgements

We gratefully thank P Kloc for providing the material data and absorption coefficients.

References